Development of a Quantum Based Microwave Power Measurement

T. P. Crowley, E. A. Donley, T. P. Heavner, J. R. Juroshek, B. F. Riddle

National Institute of Standards and Technology, Boulder, CO, 80305, USA

Abstract — **An initial proof-of-principle experiment to measure microwave power based on quantum mechanical principles is presented. An RF magnetic field causes laser-cooled cesium atoms to oscillate between two hyperfine levels at a rate proportional to the field strength. The populations vary as predicted with relative microwave intensity and duration.**

I. Introduction

The most accurate methods for measuring microwave power are based on the principle of DC substitution [1]. The primary national standards at NIST for measuring RF power are calorimeters that compare the heat produced by RF power with that produced by DC power. There are a number of transfer standards that also rely on similar principles. Because of differences in the RF and DC heat dissipation process, correction factors and efficiencies are needed to properly measure the RF power. These RF-DC differences dominate the uncertainties, which range from 0.1% to a few percent in the best cases, and tend to higher values at higher frequencies. For example, part of the transfer standard effective efficiency arises from RF heat dissipation in the input section of a sensor that is not duplicated in the DC measurement. Obtaining lower uncertainties will probably require development of techniques that do not rely on DC substitution.

Standards which are traceable directly to fundamental atomic processes have greatly increased the accuracy of measurements in areas such as DC voltage and time and frequency and led to new applications. Similar capabilities in microwave amplitude measurements could potentially revolutionize measurements and techniques.

The experiment described in this paper is a proof of principle to demonstrate that it is possible to make microwave amplitude (i.e. power) measurements based on a quantum standard. The experiments were conducted using a small cesium fountain experiment that is designed for use as a frequency standard [2]. Consequently, the apparatus is not optimized for microwave power measurements and this will affect the uncertainties that can be achieved with the present apparatus.

An initial analysis demonstrating a relative measurement is presented here. The analysis necessary to make an absolute power measurement is being performed and should be completed before the conference.

An experiment similar to the one presented here was previously performed at the National Research Council in Canada [3]. In that experiment, laser cooled rubidium atoms were exposed to a 6.8345 GHz microwave pulse outside an open-ended waveguide. The microwave radiation induced the rubidium atoms to oscillate between two quantum states. This process is known as a Rabi oscillation and the oscillation rate as the Rabi frequency. A series of measurements with different pulse lengths was made and the number of atoms to change state was then plotted vs pulse length. The Rabi frequency for a given power level was determined by fitting the data to a cosine function. This was repeated for several different power levels and the Rabi frequency was found to scale linearly with the square root of the microwave power as expected.

Our experiment's chief advantage is that the atoms are exposed to the microwave beam in an enclosed structure which makes it easier to calculate and control stray field effects. This makes absolute comparisons between conventional and quantum power measurements easier to perform.

Other differences are that we effectively pulse the field spatially instead of temporally. This is less convenient to alter and as a result our comparison relies on the linearity corrections made in the power meter connected to a diode monitor detector. The two experiments are also done with different atoms and therefore different frequencies.

II. Apparatus

A. Fountain Apparatus

A diagram of the cesium fountain apparatus is shown in Fig. 1. A collection of about 10^8 cesium atoms at a temperature of a few μK is formed in 300 ms using a magneto-optical trap and optical molasses. The trapping is achieved with a set of six laser beams, two for each dimension. The atoms are evenly spread among the $|F=4,m_F = -4$ to $4\rangle$ states. They are launched vertically by detuning the two vertical laser beams. The trap is then turned off and the atoms exhibit a fountain motion under the influence of gravity.

A rectangular state selection cavity operating in the

US Government Work Not Protected by US Copyright

TE_{104} mode is used to select only those atoms with $m_F = 0$. The frequency and amplitude of the field in this cavity is adjusted to produce a resonant interaction in which |F=4,$m_F = 0$> atoms are transferred to the |F=3,$m_F = 0$> state. This represents one half cycle of a Rabi oscillation. The remaining atoms in the |F=4,$m_F \neq 0$> states are then removed with |F=4,m_F> ⇒ |F=5,m_F> light in the detection region.

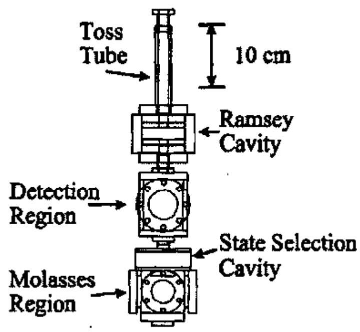

Fig. 1. Cesium fountain vacuum chamber

The atoms then pass through the Ramsey cavity where the microwave power measurement is made. The Ramsey cavity is cylindrical and operates in the TE_{011} mode. Along the axis of the cavity, the ideal magnetic field is given by $H = H_0 \sin(\pi z/d)$ where z=0 is at the bottom of the cavity and d is the cavity height. Large diameter apertures (1.4 cm) in the center of the endcaps are designed to allow as many of the atoms to pass through the cavity as possible. Thus, signal level is increased at the expense of perturbations to the field structure. The mode is excited with one of two loop antennas (~ 3 mm diameter) located opposite each other in the cavity midplane.

The microwave signal at 9.192 GHz is resonant with the |F=3, $m_F = 0$> to |F=4, $m_F = 0$> transition between two hyperfine levels of the cesium ground state. This resonance causes the atoms to oscillate between the two hyperfine levels at the Rabi frequency which is given by:

$$\Omega_R = \frac{\mu_B g_j M H}{\hbar} \quad (1)$$

where μ_B is the Bohr magneton, $g_j = 2$ is the g factor, M is the quantum mechanics matrix element <F,m_F |Jz| F+1,m_F>, H is the amplitude of the RF magnetic field, and $\hbar$ is Plank's constant divided by 2π. Ideally, the fraction of atoms expected in the |F=4> state after passing through the cavity is given by:

$$R = \frac{1 - \cos(\Omega_{R,eff} t_{cav})}{2} \quad (2)$$

where t_{cav} is the time spent in the Ramsey cavity and the effective Rabi frequency, $\Omega_{R,eff}$, is used to account for the variation in magnetic field strength with cavity position. A numerical calculation of the RF field structure in the cavity (including aperture and stray field effects) will be used to evaluate the ratio $\Omega_{R,eff}/\Omega_{R,max}$ where $\Omega_{R,max}$ is the Rabi frequency at the peak field location. In the ideal case, the ratio is $2/\pi$.

The amount of time spent in the cavity is set by the initial launch velocity (or alternatively, the toss height). The RF magnetic field strength in the cavity is controlled with a variable attenuator. With the apparatus in this experiment, modifying the field strength is more easily achieved than changing the toss height since changing the toss height requires changing the timing of the detection apparatus. In addition, the field strength can be changed over a wider dynamic range.

After the atoms pass through the Ramsey cavity, the microwave power to the cavity is turned off by switching in a large attenuation. Thus, the atoms are not exposed to any further microwave radiation as they complete the fountain motion and fall back down. They therefore stay in the same quantum state until they reach the detection region. There they encounter |F=4, m_F> ⇒ |F=5, m_F> light. Scattered photons of this light are detected and yield a signal proportional to the number of |F=4, m_F> atoms. The light also removes the |F=4> atoms. The |F=3> atoms that remain then encounter a repump laser which puts them into the |F=4> state and they are detected in the same manner. The fraction of atoms in the |F=4> state, $R = N4/(N3+N4)$ where $N3$ and $N4$ are the number of atoms in each state is calculated for each run and stored in a data file. Each data point we present has then been averaged over 21 nearly identical runs.

B. Microwave Measurement Apparatus

Microwave power is also measured by conventional techniques using the apparatus illustrated in the Fig. 2. The diode detector is used to monitor the power at all times. A power standard is used to calibrate the ratio of the power leaving port 3 to the diode monitor reading The attenuator immediately after the source is used to vary the power delivered to the Ramsey cavity and diode.

The incident power leaving port 3 of the coupler during the main experiment is then given by:

$$P_{inc,3} = P_{inc,3s} \frac{P_2}{P_{2s}} \frac{|1 - \Gamma_g \Gamma_s|^2}{|1 - \Gamma_g \Gamma_r|^2} \quad (3)$$

where Γ_g is the equivalent generator reflection coefficient of the coupler, Γ_s and Γ_r are the reflection coefficients of the standard and the combined Ramsey cavity and transmission lines, P_2 is the power measured at the diode, P_{2s} is the diode power measured when the standard was attached and $P_{inc,3s}$ is the incident power during the standard measurement.

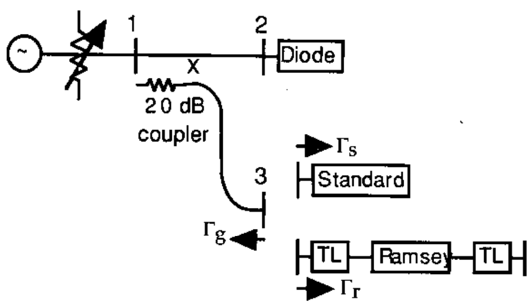

Fig. 2. Microwave measurement arrangement. TL = Transmission Line, other items identified in text.

Due to its high Q, the microwave power level needed in the Ramsey cavity to achieve one complete Rabi oscillation cycle is less than 50 nW. Therefore, the system was arranged so that a diode monitor sensor was connected to the main signal coming from port 2 of a 20 dB coupler with the cavity connected to the smaller signal from port 3. The diode sensor is a commercial detector with a dynamic range of 100 pW to 100 mW. Linearity is maintained over this range by switching between diode stacks.

Because the experiment was not designed with microwave power measurements in mind, the only access to the Ramsey cavity is through approximately 50 cm long semi-rigid cables represented by the "TL" sections in Fig. 2. In addition, the semi-rigid cable connectors are too far from a feasible VNA location to be reached with standard cables.

All reference planes had 3.5 mm connectors. Any extra Type A uncertainty due to the non-standard VNA cables can be estimated based on multiple connect measurements that were made (including some in which the cables were moved). In order to make an absolute comparison between the Rabi oscillation measurement and these power measurements, we need to account for losses in the semi-rigid transmission lines. We hope to separate out that loss from the power absorbed in the microwave cavity by comparing VNA measurements off resonance with those on resonance.

III. MEASUREMENTS

Data was taken for toss heights of 32 and 45 cm. This represents the approximate upper and lower limits for the experiment. Since this means we only have two values of t_{cav} in equation (2), we demonstrate the Rabi oscillations by varying the power using the variable attenuator shown in Fig. 2. In Fig. 3(a), we show the ratio $R = N4/(N3+N4)$ (from equation (2)) plotted versus $sqrt(P_2)$, the square root of the diode power. Rabi oscillations are clearly observed for both toss heights, including four cycles for the 32 cm toss height data. The amplitude of the peaks decreases with power level. This could be due to the finite size of the atom cloud which encounters magnetic fields of differing intensities, although we have not definitively established that connection. The ability to fit a cosine function to the data indicates that the Rabi oscillation response varies as expected with power.

In order to compare the 45 and 32 cm toss height data, we replot the data in Fig. 3(b) with an abscissa proportional to $sqrt(P_2)t_{cav}$. Since P_2 is proportional to H^2, this is proportional to the argument of the cosine in equation (2) and both sets of data should follow the same curve. Both sets were fit to functions of the form, $y = a + b\ cos(f\ sqrt(P_2)\ t_{cav})$ for the first two cycles. The key parameter obtained from the fit is the variable f. The values obtained are 2.868×10^5 for the 32 cm toss and 2.820×10^5 for the 45 cm toss. The agreement within 2% is believed to be within uncertainty in the measurements and indicates that the relative scaling of the Rabi oscillation with exposure time is also valid.

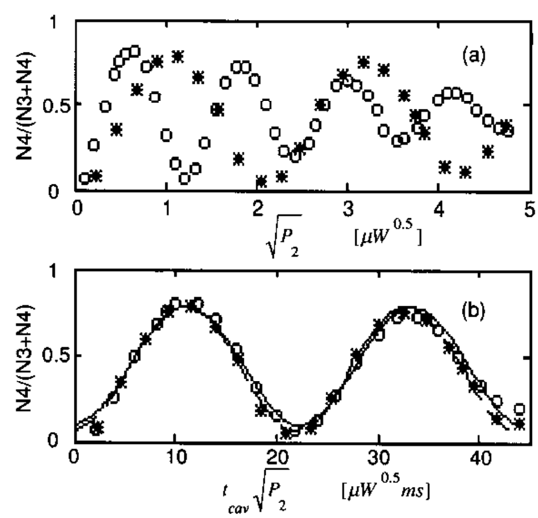

Fig. 3. Fraction of atoms in |F=4> state after undergoing Rabi oscillations due to microwave radiation. (o and solid line) - 32 cm toss data. (* and dashed line) - 45 cm toss data.

IV. Future Work

The next stage in this work, expected to be completed within the next few months is an absolute comparison of the conventional microwave measurements with Rabi oscillation measurements. By using the VNA measurements, modeling of the cavity, and some approximations, we should be able to take the magnetic field strength measured by the Rabi oscillation experiment and use it to calculate an expected power reading for a standard on port 3. This number will be obtained independently from the standard measurement. We plan on presenting these results at the conference.

A longer range goal is to provide a comparison between the conventional technique and the quantum technique using an apparatus designed for this purpose so that lower uncertainties in the comparisons are feasible.

Acknowledgments

The authors would like to thank S. R. Jefferts for suggesting the use of the fountain apparatus for this experiment. The VNA measurements were performed by D. LeGolvan. Helpful discussions with M. Janezic and other NIST staff are also appreciated.

References

[1] A. Fantom, "*Radio Frequency and Microwave Power Measurement*", London: Peter Peregrinus Ltd., 1990.

[2] T. P. Heavner, D. M. Meekhof, D. Kujundzic, and S. R. Jefferts, "Development of a miniature laser-cooled Cs fountain frequency standard at NIST", *Proceedings of the 2000 IEEE/EIA International Frequency Control Symposium and Exhibition,* pp. 711-713, June 2000.

[3] D. Paulusse, N. Rowell and A. Michaud, "Realization of an atomic microwave power standard", *2002 Conference on Precision Electromagnetic Measurements, Conference Digest,* pp. 194-195, June 2002.